Downside Risk analysis applied to Hedge Funds universe

Josep Perelló

Departament de Física Fonamental, Universitat de Barcelona, Diagonal, 647, 08028-Barcelona, Spain

Abstract

Hedge Funds are considered as one of the portfolio management sectors which shows a fastest growing for the past decade. An optimal Hedge Fund management requires a high precision risk evaluation and an appropriate risk metrics. The classic CAPM theory and its Ratio Sharpe fail to capture some crucial aspects due to the strong non-Gaussian character of Hedge Funds statistics. A possible way out to this problem while keeping CAPM simplicity is the so-called Downside Risk analysis. One important benefit lies in distinguishing between good and bad returns, that is: returns greater (or lower) than investor's goal. We study several risk indicators using the Gaussian case as a benchmark and apply them to the Credit Suisse/Tremont Investable Hedge Fund Index Data.

Key words: econophysics, Hedge Funds, Downside Risk, CAPM
PACS: 89.65.Gh, 02.50.-r, 02.70.Rr, 05.45.Tp

1 Introduction

Hedge Funds are considered as one of the portfolio management sectors which shows a fastest growing for the past few years [1,2]. These funds have been in existence for several decades but they do not have become popular until the 1990's. It is said that Hedge Funds are capable of making huge profits but sometimes we get some news announcing that a certain Hedge Fund suffered spectacular losses. Due to their at least apparent high and unpredictable fluctuations, it is necessary to keep the risks we take when we trade with Hedge Funds under rigorous control [3,4].

Email address: `perello@ffn.ub.es` (Josep Perelló).
URL: `http://www.ffn.ub.es/pages/perelloen.html` (Josep Perelló).

The Capital Asset Pricing Model (CAPM) is the classic method for quantifying the risk of a certain portfolio [5,6]. Basically, the so-called Ratio Sharpe [6] evaluates the quality of a certain asset by normalizing the asset growth expectation with the volatility. Thus, based on the fact that the asset growth expectation must be high and volatility low, a *good* Hedge Fund holds a high Ratio Sharpe. And the better the Hedge Fund the more attractive and advisable is to invest in this fund. Hedge Fund managers begin to trade with an specific Hedge Fund only when this fund gets an annual Ratio Sharpe approximately greater than one [1]. Up to this point the fund can provide benefits after removing trading costs.

However, CAPM theory is sustained under the hypothesis that underlying assets are Gaussian distributed where one only needs to know the mean and the variance of returns. As it has been observed this appears to be an unrealistic scenario in financial markets [7,8,9,10] with important implications in risk analysis and its mean-variance framework (see for instance [11]). The situation is much more dramatic in the Hedge Fund universe since these funds are clearly non-Gaussian having wild fluctuations and strong asymmetries in price changes. Indeed, these funds are characterized by their big sensitivity to the market crashes and by trading with products such as derivatives showing a pronounced skewness in their price distribution. For instance, a very well-known CTA Hedge Fund had a poor Ratio Sharpe (0.19) but, despite this mediocre mark, their earnings during the 2000 raised beyond the 40% [1]. Conversely, after 31 months of trading, the famous fund of LTCM had a very appealing ratio (4.35) and nothing seemed to forecast and announce its posterior debacle [1]. These two examples are not very exceptional cases, and they make us reexamine the validity of the CAPM theory. Everything seems to indicate that the CAPM method is not complete enough for evaluating the risks involved in the Hedge Fund management.

Our aim here is to explore some alternatives in the context of the so-called Downside Risk analysis [1,11,12]. We will have a look on some of the risk indicators presented in the literature, provide new results related to these risk measures and finally make some empirical measurements in Credit Suisse/Tremont Investable Hedge Fund Index Data. The paper is therefore structured as follows. Section 2 briefly describe data set used for the Downside Risk indicators. The next section is devoted to present the backgrounds of the Downside Risk approach. Afterwards we present the Adjusted Ratio Sharpe in Section 4, the Sortino ratios in Section 5, and the Gain-Loss Ratio is left to Section 6. Section 7 provides few conclusions and the equivalence between the Omega function and the Gain-Loss Ratio is shown in Appendix A.

2 The Hedge Fund data set

There are several third-party agencies that collect and distribute data in Hedge Fund performance [1]. For this paper, we have used the data supplied by the Credit Suisse/Tremont (CST) Index LLC [13]. This company is a joint venture between Credit Suisse and Tremont Advisers Inc. The Credit Suisse and Tremont have combined their resources to set several benchmarks for a large collection of Hedge Fund strategies. They provide a master index and series of sub-indices that represent the historical returns for different Hedge Fund trading styles [1,13].

The weight of each fund in an index is given by the relative size of its assets under management. This makes the CST Index the first asset-weighted indices in the industry. Asset-weighting, as opposed to equal-weighting, provides a more accurate depiction of an investment in the asset class. In addition, CST has a web site [13] that provides an up-to-date and historical information that allows the user to select and download data. Information available is public. The selection of funds for the Credit Suisse/Tremont indices is done every quarter. The process starts by considering all 2,600 US and offshore Hedge Funds contained in the TASS database, with the exception of funds of funds and managed accounts.

In the present case, we have analyzed the monthly data for these indices during the period between 31st December of 1993 until the 31st January of 2006. This period corresponds to 145 data points for each Hedge Fund style. This is not a huge amount of data but it is enough to perform a reasonably fair and reliable statistical estimation of the quantities we here deal with. In Fig. 1 we show the indices that were all normalized to 100 at the beginning of 1994. We also show the monthly logarithmic return change $R_\Delta(t) = \ln(S(t+\Delta)/S(t)$ where $S(t)$ is current price index and Δ is one month. Table 1 shows us how the mean-variance framework fails to explain the statisitcs of the majority of Hedge Fund styles monthly returns. Kurtosis can raise to values larger than 20 and skewness is usually negative and may take values larger than 3.

3 The Downside Risk Metrics: Main definitions

For the reasons mentioned above, the so-called Downside Risk analysis has been gaining wide acceptance in recent years [3,4,11]. One important benefit of Downside risk lies in distinguishing between *good* and *bad* returns: Good returns are greater than the goal, and bad returns are the ones below the goal. Downside risk measures incorporates an investor's goal explicitly and

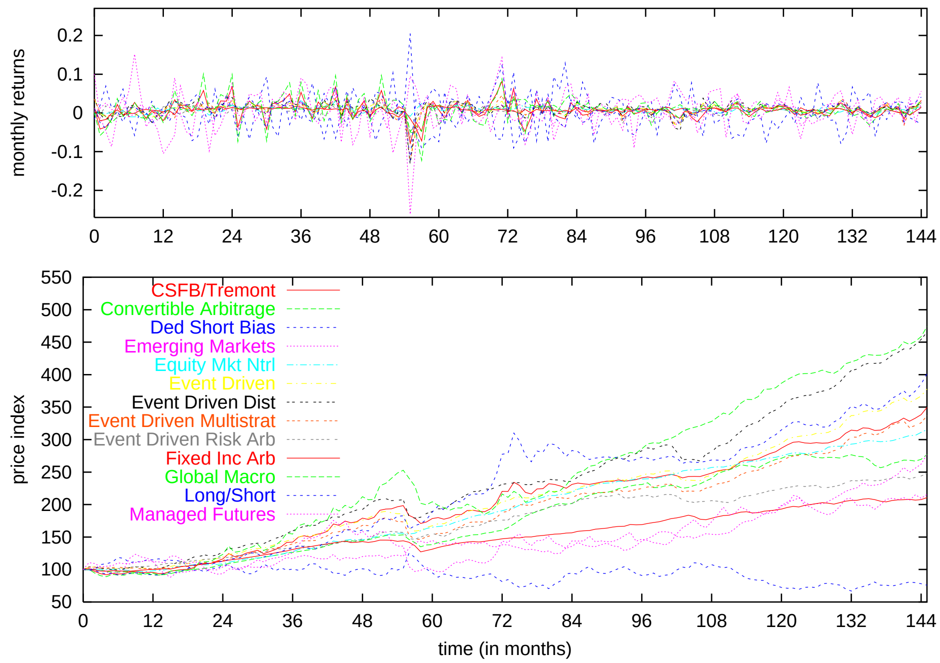

Fig. 1. The price and the monthly return change time series for the Credit Suisse/Tremont (CST) Index and subindices during the period between 31st December of 1993 until the 31st January of 2006.

defines risk as not achieving the goal. In this way, the further below the goal, the greater the risk. And, in the opposite side, returns over the goal does not imply any risk. Within this approach, a portfolio's riskiness may be perceived differently by investors with different goals. This is perhaps more realistic than the CAPM theory approach where all investors have the same risk perception with the Ratio Sharpe.

In the present work, we mainly relate the target return to the mininum acceptable return for considering profitable the trading operation. And statistical risk would be associated with the unsuccesful tentatives of obtaining a higher return than the target return. However, the target return could also be related to the maximum loss that a Hedge Fund can afford measuring risk in a somewhat similar way as the Value at Risk measures do [14].

We consider the price of the asset S at time t and its initial price S_0 at time $t = 0$. Let us thus define the Excess Downside as:

$$D(R,T) = \max[0, T - R] = \begin{cases} T - R \text{ if } T > R, \\ 0 \text{ if } T \leq R; \end{cases} \qquad (1)$$

where $R \equiv \ln(S/S_0)$ is the subsequent return change and T is the target return. Observe that the profile of the Excess Downside is identical to the payoff of

Table 1
Main statistical values for the whole set of Hedge Fund style indices during the period between 31st December of 1993 until the 31st January of 2006. We show the first moment, the standard deviation, the kurtosis and the skewness for the monthly returns. Most of the indices have a kurtosis larger than one and some of them also have a non negligible skewness. The mean-variance framework might fit well only for very few of them (the Managed Futures and the Equity Market Neutral styles).

Hedge Funds indices	average	std dev	kurtosis	skewness
Credit Suisse/Tremont Index	0.008688	0.02255	2.335	-0.03966
Convertible Arbitrage	0.007074	0.01383	3.180	-1.367
Dedicated Short Bias	-0.001919	0.04913	1.239	0.6276
Emerging Markets	0.007086	0.04817	6.519	-1.142
Equity Market Neutral	0.007977	0.008410	0.3435	0.2986
Event Driven	0.009229	0.01680	27.82	-3.773
Fixed Income Arbitrage	0.005173	0.01097	17.81	-3.243
Global Macro	0.01080	0.03179	3.033	-0.2113
Long/Short	0.009653	0.02942	3.998	-0.04185
Managed Futures	0.005327	0.03456	0.4723	-0.09699
Event Driven Distressed	0.01068	0.01896	22.24	-3.255
Event Driven Multistrategy	0.008430	0.01795	19.28	-2.855
Event Driven Risk Arbitrage	0.006306	0.01217	7.221	-1.395

the European put option [14]. We can study the first and second moments of the Excess Downside $D(T)$. Therefore, the first moment is defined as

$$\mu_-(T) \equiv E\left[D(R,T)\right] = \int_{-\infty}^{T} (T-R)p(R)dR, \tag{2}$$

while second moment reads

$$d(T)^2 \equiv E\left[D(R,T)^2\right] = \int_{-\infty}^{T} (T-R)^2 p(R)dR. \tag{3}$$

The square root of second moment (3) is also called Excess Downside Deviation (EDD).

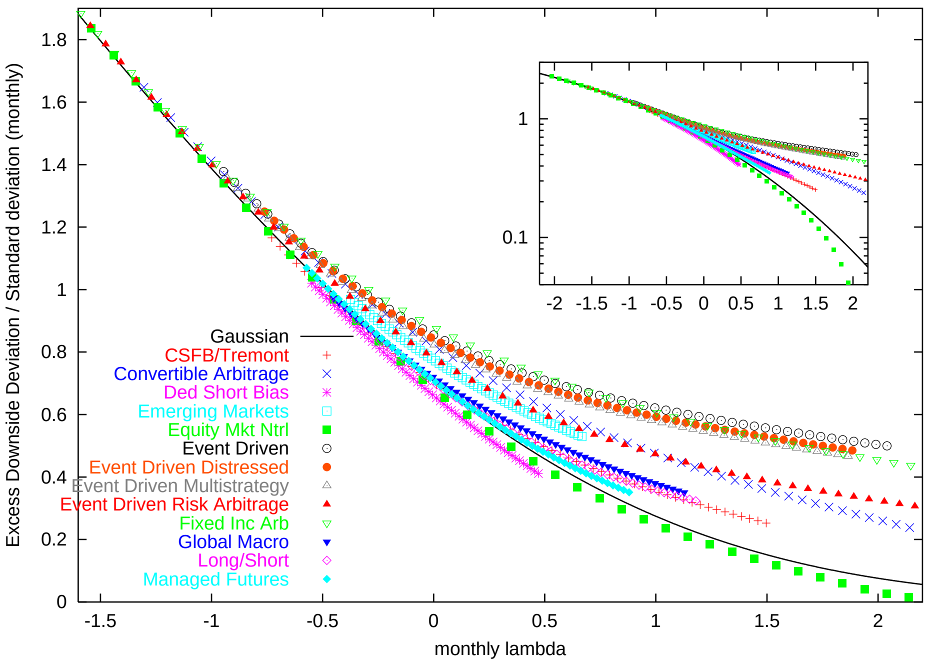

Fig. 2. The quotient between the Excess Downside Deviation and the Standard Deviation for the Credit Suisse/Tremont (CST) Index and subindices during the period between 31st December of 1993 until the 31st January of 2006. We show the quotient given by Eq. (7) in terms of lambda given by Eq. (6) for several Hedge Fund styles when target return is between $T = -30\%$ and $T = 30\%$ annual rates. The inset provides the same numbers but in a logarithmic scale.

4 The Adjusted Ratio Sharpe

First possible extension wants to keep the CAPM approach but with a rough correction based on the empirical EDD. This is probably a simplest sophistication to the mean-variance framework. Its interest is based on the fact that correction wants to replace volatility σ by $d(T)$. The CAPM measures risk of a certain asset with the well-known

$$\text{Ratio Sharpe} = \frac{\mu - r}{\sigma} \tag{4}$$

where r is the risk-free interest rate ratio, $\mu = E[R]$ is the first moment of the return, and $\sigma^2 = \text{Var}[R]$ is the return variance.

Johnson etal. [15] propose an Adjusted Ratio Sharpe as “the Ratio Sharpe that would be implied by the fund’s observed Downside Deviation if returns were distributed normally”. We thus assume that returns were Gaussian

$$p(R) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(R-\mu)^2}{2\sigma^2}\right], \tag{5}$$

and define the modified Ratio Sharpe with the quotient

$$\lambda \equiv \frac{\mu - T}{\sigma}. \tag{6}$$

This variable is very important not only in this rough correction of the Ratio Sharpe but also in the analysis of the future risk indicators we will show herein. These measures can be all represented in terms of lambda if underlying is Gaussian.

Therefore, from the Excess Downside Deviation (3) we can write

$$\frac{d(T)^2}{\sigma^2} = \left(\lambda^2 + 1\right) N(-\lambda) - \lambda N'(\lambda). \tag{7}$$

where N is the probability function

$$N(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\lambda} \exp\left(-\frac{y^2}{2}\right) dy, \tag{8}$$

while its derivative is denoted by a prime and reads $\exp\left(-\lambda^2/2\right)/\sqrt{2\pi}$. Figure 2 plots the quotient in terms of the modified lambda for a broad range of target returns T, from -30% until 30% annualized rates.

We can finally numerically invert the ratio $d(T)/\sigma$ obtained from Eq. (7). We will thus obtain the so-called Adjusted Ratio Sharpe. That is: the lambda that corresponds to the Excess Downside Deviation ratio in the event of returns are Gaussian and accomplishing the equality (7). We show the resulting empirical results in Table 2 for the special case when $T = 5\%$. The Adjusted Ratio Sharpe might differ significantly from the Ratio Sharpe.

5 The Sortino and Upside Potential ratios

More dramatic modifications of the Ratio Sharpe are the ones that Sortino etal. propose in their works [16,17,18,19]. In contrast with the adjusted Ratio Sharpe, this pair of indicators would be self-consistent measures that do not need to assume that the underlying asset is Gaussian. All averages involved in these indicators are directly computed from the historical data. However, these two new ratios look like the Ratio Sharpe. In both cases, the risk-free rate is replaced by the target return and the volatility by the Excess Downside Deviation.

Table 2
The monthly Adjusted Ratio Sharpe for several Hedge Fund indices. We first derive the monthly Excess Downside Deviation ratio $d(T)/\sigma$ for several Hedge Fund indices when annual return growth is $T = 5\%$. Once we get these quantities we numerically invert Eq. (7). We thus compare these results with the Ratio Sharpe.

Hedge Funds	$d(T)/\sigma$	Adj. Ratio Sharpe	Ratio Sharpe
Credit Suisse/Tremont Index	0.609	0.63	0.188
Convertible Arbitrage	0.729	-0.13	0.289
Dedicated Short Bias	0.735	-0.16	-0.0704
Emerging Markets	0.746	-0.23	-0.00229
Equity Market Neutral	0.447	1.82	0.549
Event Driven	0.756	-0.29	0.249
Fixed Income Arbitrage	0.845	-0.80	0.115
Global Macro	0.626	0.51	0.216
Long/Short	0.612	0.61	0.171
Managed Futures	0.688	0.12	0.0284
Event Driven Distressed	0.725	-0.11	0.287
Event Driven Multistrategy	0.740	-0.20	0.194
Event Driven Risk Arbitrage	0.705	0.01	0.211

First tentative is the so-called Sortino Ratio (SoR). It is defined as follows:

$$\mathrm{SoR}(T) = \frac{\mu - T}{d(T)}, \tag{9}$$

where $d(T)$ is given by Eq. (3). There is a sophistication made by the same Sortino that only wants to give reason of the excess return. This new measure is interested in the return average up to the target return. Thus, the upside potential ratio (UPR) is defined

$$\mathrm{UPR}(T) = \frac{\mu_+(T)}{d(T)}, \tag{10}$$

where

$$\mu_+(T) = \int_T^\infty (R - T)p(R)dR, \tag{11}$$

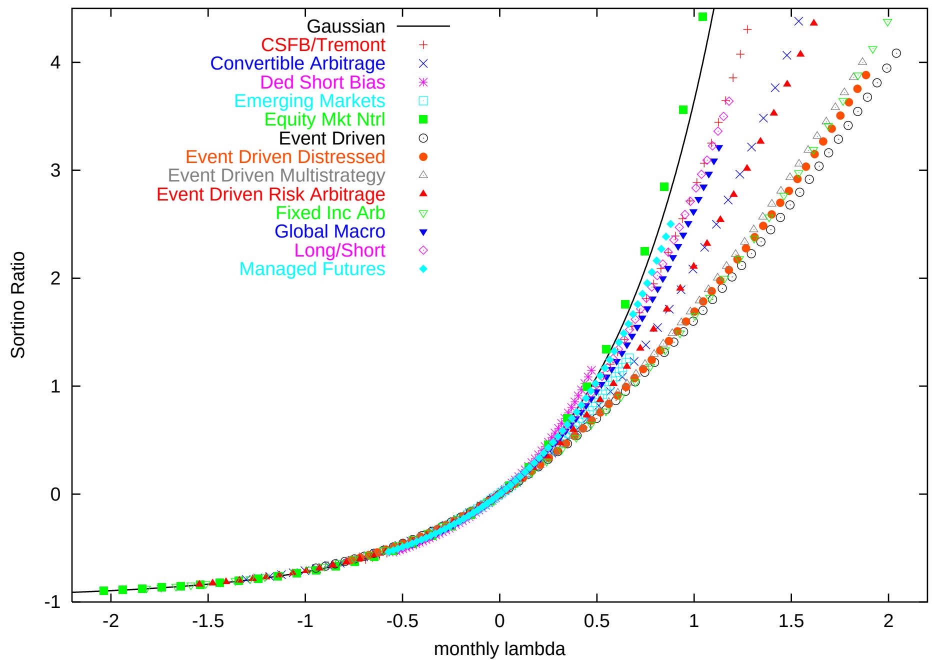

Fig. 3. The monthly Sortino Ratio for the Credit Suisse/Tremont (CST) Index and subindices during the period between 31st December of 1993 until the 31st January of 2006. We show the monthly $\mathrm{SoR}(T)$ given by Eq. (9) in terms of lambda given by Eq. (6) for several Hedge Fund styles when target return is between $T = -30\%$ and $T = 30\%$ annual rates. We compare them with the Gaussian Sortino Ratio (12) and observe that the historical data results are not very far from the Gaussian hypothesis.

or equivalently $\mu_+(T) = \mu - T + \mu_-(T)$ (cf. Eq.(2)). One therefore can also write $\mathrm{UPR}(T) = \mathrm{SoR}(T) + \mu_-(T)/d(T)$.

Both measures behave as the Ratio Sharpe. The greater ratio corresponds to the better asset. Let us calculate the quotients given Eqs. (9) and (10) in case that returns were Gaussian. We thus first need the Gaussian $d(T)$ which is already given by Eq. (7). In such a case the Sortino Ratio reads

$$\mathrm{SoR}(T) = \frac{\lambda}{\left[\left(\lambda^2+1\right) N(-\lambda) - \lambda N'(\lambda)\right]^{1/2}}, \tag{12}$$

while, also taking into account that under the Gaussian hypothesis the upside average (11) is

$$\mu_+(T) = \sigma\lambda N\left(\lambda\right) + \sigma N'(\lambda), \tag{13}$$

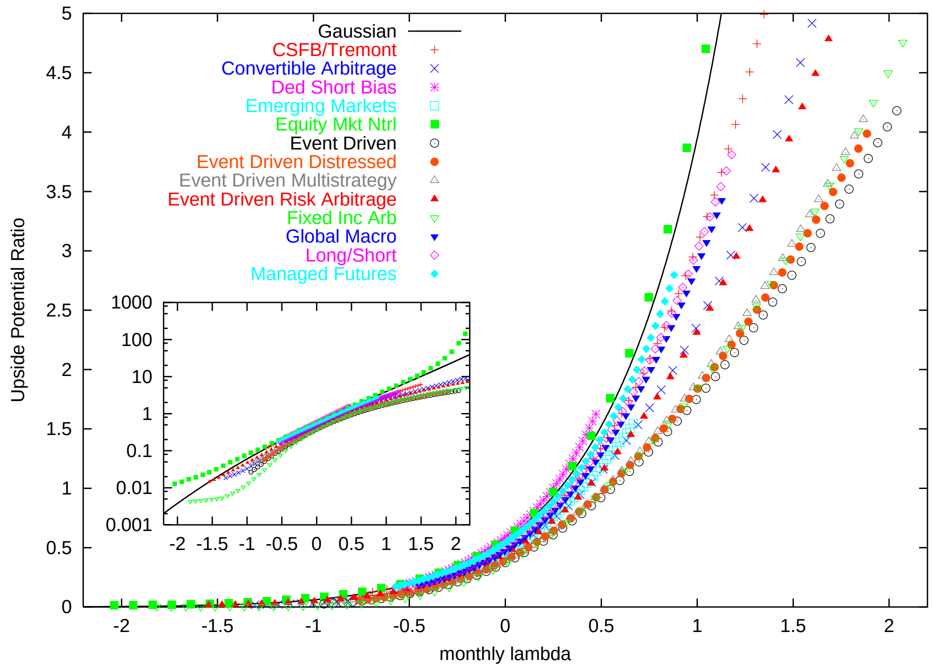

Fig. 4. The monthly Upside Potential Ratio for the Credit Suisse/Tremont (CST) Index and subindices during the period between 31st December of 1993 until the 31st January of 2006. We show the UPR(T) given by Eq. (9) in terms of lambda given by Eq. (6) for several Hedge Fund styles and when target return is between $T = -30\%$ and $T = 30\%$ annual rates. We compare them with the Gaussian UPR (14). The inset shows the same plot but with a logarithmic scale in the UPR axe.

the Upside Potential Ratio reads

$$\text{UPR}(T) = \frac{\lambda N(\lambda) + N'(\lambda)}{\left[\left(\lambda^2 + 1\right) N(-\lambda) - \lambda N'(\lambda)\right]^{1/2}}. \tag{14}$$

Note that both risk measures have been expressed in terms of the modified Ratio Sharpe

$$\lambda = \frac{\mu - T}{\sigma}.$$

Some special and limiting cases are

$$-1 < \text{SoR}(T) < \infty \qquad \text{and} \qquad 0 < \text{UPR}(T) < \infty,$$

where upper and lower bounds respectively correspond to the limiting cases $T \to \infty$ and $T \to -\infty$.

These measures could be annualized as it was done with the Ratio Sharpe. Recall that the monthly Ratio Sharpe is annualized when we multiply the ratio by the factor $\sqrt{12}$. However, in principle, this is not as easy in the present case as it has been thoroughly investigated in Ref. [17].

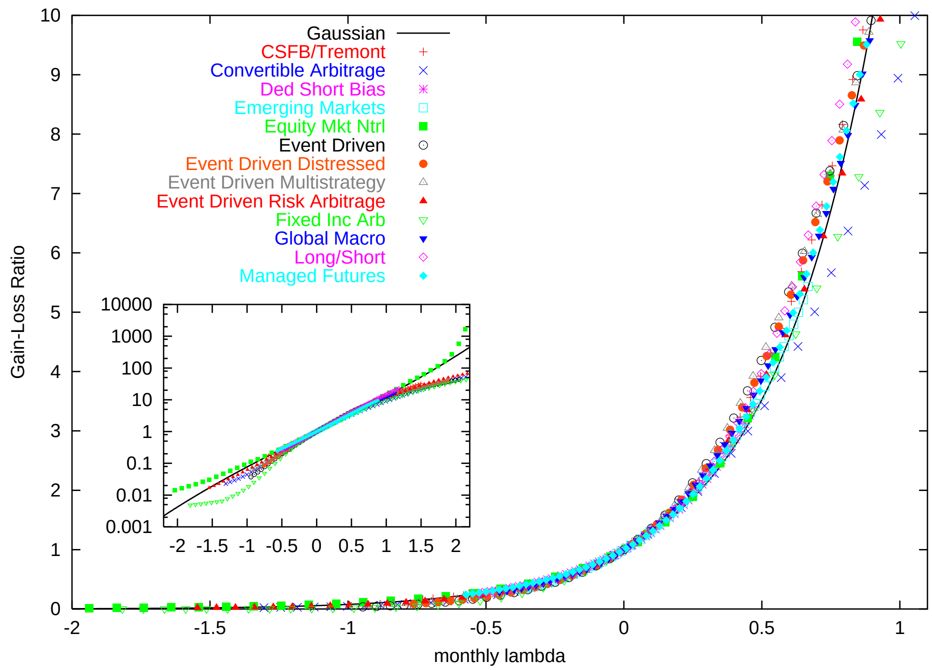

Fig. 5. The monthly Gain-Loss Ratio for the Credit Suisse/Tremont (CST) Index and subindices during the period between 31st December of 1993 until the 31st January of 2006. We show the G-L ratio given by Eq. (15) in terms of lambda given by Eq. (6) for several Hedge Fund styles and when target return is between $T = -30\%$ and $T = 30\%$ annual rates. We compare them with the Gaussian UPR (20). The inset shows the same plot but with a logarithmic scale in the Gain-Loss Ratio axe.

We represent the Sortino and Upside Potential ratios in Figs. 3 and 4. We there plot the Gaussian case as a benchmark and in terms of lambda. The empirical data is also shown in terms of lambda to get the results comparable. For the empirical data we have computed the risk measures for a broad range of target returns T, from -30% until 30% annualized rates. In general, we do not perceive much differences between the two plots. In both, the more Gaussian case and in a larger domain in terms of lambda corresponds to the Equity Market Neutral strategy. The rest of trading styles could be packed in several groups in the two risk measures. Particularly, for large λ (negative T) we can easily see that risk in most of the Event Driven indices and the Fixed Income Arbitrage index coincides. The reason why Gaussian curve is beyond most of the indices risk measures should be found in the negative and nonnegligible skewness of these indices.

6 The Gain-Loss Ratio

Bernardo and Ledoit [20] propose another risk measure called Gain-Loss Ratio (G-L). This is probably the most well-grounded measure of the existing alternatives to CAPM theory. This measure is an alternative approach to "asset pricing in incomplete markets that bridges the gap between the two fundamental approaches in finance: model-based pricing and pricing by no arbitrage". In contrast with the Sortino ratios, G-L have no comparable magnitudes with the Ratio Sharpe.

In the simplest case, the attractiveness of an investment opportunity is measured by the Gain-Loss ratio

$$\text{G-L}(T) = \frac{\mu_+(T)}{\mu_-(T)}, \tag{15}$$

which is the quotient between the averages of positive and negative parts of the payoff after removing the trading costs included in the target return T. This ratio is the basis to an alternative asset pricing framework by limiting the maximum Gain-Loss ratio. We can therefore restrict the admissible set of pricing kernels and also constrain the set of prices that can be assigned to assets. In other words, we admitt that there are arbitrage opportunities but limited in a certain range of prices. In the same way, Bernardo and Ledoit [20] state that the theoretical no arbitrage assumption is related to the mathematical demmand that the Gain-Loss Ratio is 1.

Following the notation used above we have that

$$\mu_-(T) \equiv E\left[(T-R)^+\right] = \int_{-\infty}^{T} (T-R)p(R)dR, \tag{16}$$

and

$$\mu_+(T) \equiv E\left[(R-T)^+\right] = \int_{T}^{\infty} (R-T)p(R)dR. \tag{17}$$

Note that from these definitions we can obtain the following expression

$$\mu_+(T) - \mu_-(T) = \mu - T = \lambda\sigma. \tag{18}$$

where we also take into account the definition of λ given by Eq. (6).

In case we assume that returns are Gaussian, we have already obtained the $\mu_+(T)$ average. From Eqs. (13) and (18), we thus have

$$\frac{\mu_-(T)}{\sigma} = -\lambda N(-\lambda) + N'(\lambda), \tag{19}$$

and

$$\frac{\mu_+(T)}{\sigma} = \lambda N(\lambda) + N'(\lambda).$$

Therefore, the Gain-Loss Ratio reads

$$\text{G-L}(T) = \frac{\mu_+(T)}{\mu_-(T)} = 1 + \frac{\lambda}{N'(\lambda) - \lambda N(-\lambda)}. \tag{20}$$

Note that no arbitrage corresponds to $\lambda = 0$ which means that average μ equals the target return T. Also observe that the Gain-Loss has not time units. This means that annual Gain-Loss ratio should have the same value as the monthly Gain-Loss ratio. This is a very interesting and poweful property that avoids any discussion about the way we derive the annualized risk indicator as it happens with the Sortino ratios and the Adjusted Ratio Sharpe.

The bounds of the Gain-Loss Ratio are

$$0 \leq \text{G-L}(T) \leq \infty. \tag{21}$$

which respectively correspond to $T \to \infty$ and $T \to -\infty$. One can also see that G-L ratio at least in Gaussian framework is a non decreasing function whose fastest growing is for positive lambdas, that is $T < \mu$. Thus, risk measure is very sensitive to small changes when λ is positive while for negative lambda G-L ratio does not provide a lot of information.

Figure 5 confirms this very last statement. The same plot also depicts the empirical results for a broad range of target of annualized returns between -30% and +30%. The more Gaussian behaviour and in a broader domain of lambda again corresponds to Equity Market Neutral strategy although other styles such as the Managed Futures also follows nicely the curve. In contrast with the SoR and UPR risk measures, it is much more difficult to detect groups with very similar behaviour.

Finally one should mention that the so-called Omega function [21,22] provides exactly the same measure as the Gain-Loss ratio. This is shown in Appendix A.

7 Final remarks

Hedge Funds have enjoyed increasing levels of popularity coupled with opacity and some myths [1,2]. We here have followed this recent interest in studying the Hedge Funds even from an academic purpose (see for instance Refs. [23,24] published in this journal). This is possible since some data such as the Credit Suisse/Tremont Investable Hedge Fund Index is now easily available. The current investigation wants to go one step further on the Downside risk metrics applied to the empirics of the Hedge Fund style indices. The strong non Gaussian character of financial markets have led to consider risk measures alternatives to CAPM theory in the context of the Downside Risk [1,3,4,11]. The measures are able to distinguish between good and bad returns compared to our own personal target T in a very simple manner. In particular, we have focussed on the Adjusted Ratio Sharpe, the Sortino ratios and the Gain-Loss ratio from both a theoretical and empirical point of view. We have seen that the Downside Risk framework provides quite robust measurements and it appears to be the most natural extension to the CAPM theory and its mean-variance framework.

The Hedge Funds is a field where these risk measures have most promising future. There are mainly two reasons. First reason is the existence of wild fluctuations and pronounced negative skewness in data. And secondly is that there are few empirical data points available (of the order of hundred points). This last reason makes impossible to work with other more sophisticated risk metrics which are more sensitive to the wildest fluctuations. However, we have also seen that the Gaussian results for the studied Downside risk measures are still important. We have shown that they work very well as a benchmark if we represent the empirical risk measures in terms of a modified Ratio Sharpe $\lambda = (\mu - T)/\sigma$. Perhaps quite suprisingly, we can also see in Figs. 3, 4, and 5 that the Gaussian trading investment behaviour works better than most of the sophisticated trading style indices. Main reason lies on the fact that the Hedge Fund provide high benefits with the cost of having in most cases a negative skewness. Downside risk measures take into account this asymetry and includes it to the risk perception of each investor. This is therefore another argument for using the Downside framework since Ratio Sharpe might wrongly overvaluate the quality of a Hedge Fund by ignoring the skewness (and kurtosis) effects in risk analysis.

There are many other interesting things to study under the current perspective. First possibility is to deeper study these risk indicators when returns obey another return distribution much more realistic like a Laplace distribution [25,26] or even a power law distribution. We could also compute the here presented risk measures when target return is another asset. Finally, another possibility is to go further and study the effect of these analysis in the multifactor market modelling [5,27,28,29,30]. However, these should left for future investigations.

Acknowledgements

The author acknowledges support from Dirección General de Investigación under contract FIS2006-05204.

A The Omega function

There exists another risk measure which is a different way of evaluating the Gain-loss ratio. This is perhaps more efficient since data analysis using this formalism is more reliable. The so-called Omega function is equivalent to the Gain-Loss Ratio although their authors do not tell anything about this fact [21,22]. The Finance Development Centre [31] proposes the following measure:

$$\Omega(T) = \frac{I_2(T)}{I_1(T)}, \tag{A.1}$$

where

$$I_2(T) = \int_T^{\infty} (1 - F(R))dR \qquad \text{and} \qquad I_1(T) = \int_{-\infty}^{T} F(R)dR$$

taking into account that $F(R)$ is the cumulative distribution of the probability distribution of returns R, i.e.,

$$F(R) = \int_{-\infty}^{R} p(x)dx.$$

Let us focus on the expressions for I_1 and I_2. Firstly, integrating by parts we have

$$I_1(T) = \int_{-\infty}^{T} F(R)dR = TF(T) - \lim_{R\to-\infty} RF(R) - \int_{-\infty}^{T} Rp(R)dR.$$

Due to the fact that function F comes from a probability distribution, by definition it is necessary that

$$\lim_{R\to-\infty} RF(R) = 0.$$

Moreover, we can rewrite the previous expressions as follows

$$TF(T) - \int_{-\infty}^{T} Rp(R)dR = \int_{-\infty}^{T} (T-R)p(R)dR,$$

and finally see (cf. Eq. (16)) that

$$I_1(T) = E\left[(T-R)^+\right] = \mu_-(T). \tag{A.2}$$

Secondly, we can do the same with I_2. Thus, similar calculations lead us to state that (cf. Eq. (17))

$$I_2(T) = \int_{-\infty}^{T} (1 - F(R))dR = \mu_+(T). \tag{A.3}$$

Therefore, according to the values derived for I_1 and I_2 and definition 15, we find that Omega function and Gain-Loss Ratio coincide since

$$\Omega(T) = \frac{\mu_+(T)}{\mu_-(T)} = \text{G-L}(T). \tag{A.4}$$

Obviously, Omega will have the same bounds and behavior as the Gain-Loss Ratio and previous results can be also applied to the Omega function.

The authors also define the Omega risk as

$$\Omega_R(T) \equiv \frac{\partial \ln \Omega}{\partial T} = \frac{1}{\Omega}\frac{\partial \Omega}{\partial T}. \tag{A.5}$$

This variable wants to measure the sensitivity of the Omega function with changes in the target return T. Therefore, according to the definition (A.1),

$$\Omega_R(T) = \frac{1}{I_2}\frac{\partial I_2}{\partial T} - \frac{1}{I_1}\frac{\partial I_1}{\partial T}.$$

But, from Eqs. (A.2)–(A.3) and taking into account that [32]

$$\frac{\partial I_2}{\partial T} = F(T) - 1 \qquad \frac{\partial I_1}{\partial T} = F(T),$$

we finally obtain

$$\Omega_R(T) = -\left[\frac{1}{I_2} + \left(\frac{1}{I_1} - \frac{1}{I_2}\right) F(T)\right].$$

Assuming that returns are Gaussian and recalling that $\lambda = (\mu - T)/\sigma$, we have that

$$\Omega_R(T) = \left(\frac{1}{\mu_-} - \frac{1}{\mu_+}\right) N(\lambda) - \frac{1}{\mu_-},$$

since μ_- is given by Eq. (19), μ_+ is given by Eq. (13),

$$\frac{\partial \mu_-}{\partial \lambda} = -\sigma N(-\lambda) \qquad \text{and} \qquad \frac{\partial \mu_+}{\partial \lambda} = \sigma N(\lambda).$$

The Omega risk is always negative since the function Ω is a non-decreasing function.

Finally, we should mention that linear transformations such as

$$T \to \phi(T) = AT + B$$

have the following Omega transformation

$$\Omega \to \begin{cases} \hat{\Omega}\left[\phi(T)\right] = \Omega(T) & \text{if } A > 0; \\ \hat{\Omega}\left[\phi(T)\right] = 1/\Omega(T) & \text{if } A < 0. \end{cases}$$

This is also true for the Gain-Loss Ratio equivalent risk indicator.

References

[1] F.-S. Lhabitant, Hedge Funds: Myths and Limits, Wiley, Chichester, 2002.

[2] L.A. Seco, Hedge funds: Truths and Myths, Revista de Economia Financiera 6 August, 2005. Downloable from: http://www.risklab.ca/seco/Seco-HedgeFunds-TruthsMyths.pdf

[3] H. Till, Quantitative Finance 2 (2002) 237–238.

[4] H. Till, Quantitative Finance 2 (2002) 409–411.

[5] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments, John Wiley, New York, 1956 and reprinted by Blackwell, Oxford, 1991.

[6] W.F. Sharpe, J. Finance 19 (1964) 425-442.

[7] R. N. Mantegna, and E.H. Stanley, Nature 376 (1995) 46-49.

[8] J. Masoliver, M. Montero, and J. M. Porrà, Physica A 283 (2000) 559-567.

[9] J. Perelló and J. Masoliver, Physica A 314 (2002) 736-742.

[10] J. Perelló and J. Masoliver, Physica A 308 (2002) 420-442.

[11] J. Estrada, The European Journal of Finance, 10 (2004) 239-254.

[12] F. Sortino, Managing Downside Risk in Financial Markets, Butterworth-Heinemann, 2001.

[13] For further information: www.hedgeindex.com

[14] P. Wilmott, Derivatives, Wiley, New York, 1998.

[15] Johnson, D., N. Macleod and C. Thomas, AIMA Newsletter September (2002) 14-16.

[16] F. Sortino, R. van der Meer, Journal of Portfolio Management 17 Summer (1991) 27-31.

[17] F. Sortino and H. Forsey, Journal of Portfolio Management Winter (1996) 35-42.

[18] F. Sortino, R. van der Meer, and A. Plantinga, Journal of Portfolio Management Fall (1999) 50-58.

[19] Ch. Pedersen and S. Satchell, Quantitative Finance 2 (2002) 217-223.

[20] A. Bernardo and O. Ledoit, Journal of Political Economy 108 (2000) 144-172.

[21] C. Keating, W.F. Shadwick, *A universal performance measure*, Finance Development Centre, working paper.

[22] C. Keating and W. F. Shadwick, AIMA Newsletter April (2002).

[23] M.A. Miceli and G. Susinno, Physica A 344 (2004) 95-99.

[24] N. Nishiyama, Physica A 301 (2001) 457-472.

[25] C. Schmidhuber and P-Y Moix, AIMA Newsletter September (2001).

[26] C. Schmidhuber and P-Y Moix, AIMA Newsletter December (2001).

[27] J. Cochrane, New facts in finance, *Economic perspectives* Federal Reserve Bank of Chicago XXIII Third Quarter (1999) 36-58.

[28] J. Cochrane, Portfolio advice for a multifactor world, *Economic perspectives*, Federal Reserve Bank of Chicago, XXIII Third Quarter (1999) 59-78.

[29] J. Cochrane, Asset pricing, Princeton University Press, Princeton, 2001.

[30] C. Low, Semidimensional risks in the cross section of stock returns, Yale University and National University of Singapore, working paper. DOI: 10.2139/ssrn.246510.

[31] www.FinanceDevelopmentCentre.com.

[32] Authors of the Finance Development Centre give these expressions. These can be different depending on the calculus convention taken.